PHYS-434 – Physics of photonic semiconductor devices, Raphaël Butté

Solution 11 – Some basic notions on laser diodes I

Exercise I: Optical resonator and lasing oscillation threshold

1. To solve this question, the treatment is similar to the case of the Fabry-Perot cavity. The main differences reside in the presence of an amplification coefficient $\exp[(\gamma - \alpha_p)d/2]$ and the absence of an angular component for the dephasing term as we consider a complex scalar plane wave which propagates under normal incidence.

   The incoming light field E_0 is such that: $E_0 = E_i e^{i(\omega t - kx)}$ ($x = 0$ at the cavity entrance).

   The first part of the transmitted field is given by:

   $$E_{1t} = E_i e^{i(\omega t - kd)} t_1 t_2 \exp[(\gamma - \alpha_p)d/2]$$

   The second part of the transmitted field, which has performed a round-trip with respect to E_{1t}, is given by:

   $$E_{2t} = E_{1t} \times e^{-2ikd} r_1 r_2 \exp[(\gamma - \alpha_p)d]$$

   Using a similar scheme, we obtain:

   $$E_{Nt} = E_{1t} \times e^{-2(N-1)ikd} (r_1 r_2)^{N-1} \exp[(N-1)(\gamma - \alpha_p)d]$$

   Then $E_t = E_{1t} + E_{2t} + ... + E_{Nt}$ which leads to:

   $$E_t = E_i e^{i(\omega t - kd)} t_1 t_2 \exp[(\gamma - \alpha_p)d/2][1 + e^{-2ikd} r_1 r_2 \exp[(\gamma - \alpha_p)d] + ...]$$

   The summation of this series can be summarized as:

   $$E_t = E_i e^{i(\omega t - kd)} t_1 t_2 \exp[(\gamma - \alpha_p)d/2] \left[\frac{1 - \left(e^{-2ikd} r_1 r_2 \exp[(\gamma - \alpha_p)d]\right)^N}{1 - e^{-2ikd} r_1 r_2 \exp[(\gamma - \alpha_p)d]} \right] (\text{with } N \to \infty)$$

2. The electromagnetic field diverges when the denominator of previous equation is equal to zero: this is the lasing oscillation condensation threshold.

   It is thus seen that two conditions must be satisfied to get lasing oscillations: a condition on the optical gain and a condition on the phase.

   Condition on the optical gain

   The optical gain of the amplifying medium must overcome the various losses occurring in the cavity: mirror transmissivity, light scattering, parasitic absorption, etc. which is summarized by the inequality: $|r_1 r_2| \exp[(\gamma - \alpha_p)d] > 1$.

   There is a lasing oscillation threshold γ_{thr} above which the gain medium will spontaneously oscillate. This threshold is given by: $\gamma_{\text{thr}} = \alpha_p - (1/d) \ln |r_1 r_2|$, which leads to:

   $$\gamma_{\text{thr}} = \alpha_p - \frac{1}{2d} \ln(R_1 R_2)$$

   when introducing the reflectivity of the mirrors.

   It is seen that the gain is damped once crossing the threshold since the equation of the transmitted field E_t indicates that an increasing γ (with $\gamma > \gamma_{\text{thr}}$) would lead to a divergence of E_t which is a physical nonsense.

3. Losses in the system for a single pass are such that:

$$\text{losses} = 1 - \exp(-\gamma_{\text{thr}} d) = 1 - \exp\left(-\frac{t_{\text{single pass}}}{\tau_{cav}}\right)$$

Thus $\gamma_{\text{thr}} d = t_{\text{single pass}}/\tau_{cav}$ and as $t_{\text{single pass}} = d/c'$, where $c' = c/n_{\text{op}}$ is the velocity of the wave propagating in the cavity, we get $\tau_{\text{cav}} = 1/(\gamma_{\text{thr}} c')$ and finally we get:

$$\tau_{\text{cav}} = \frac{n_{\text{op}}}{c\left(\alpha_p - \frac{1}{2d}\ln(R_1 R_2)\right)}$$

4. Condition on the phase

The condition on the phase which ensures that the denominator of E_t is equal to zero is such that: $kd - \varphi = n\pi$ with $n \in N$. As we consider a cavity surrounded by metallic mirrors ($\varphi = \pi$), $k_n = (n+1)\pi/d$, or, $k_m = m\pi/d$ with $m \in N^*$. Note that $k_m = 2\pi n_{\text{op}}/\lambda_m$ with $\lambda_m = c/\nu_m$, the amplified modes are given by:

$$\nu_m = \frac{mc}{2n_{\text{op}} d}$$

where ν_m values lie within the gain spectrum of the amplifying medium.

The splitting between each mode is equal to $c/(2n_{\text{op}} d)$.

The number of allowed amplified modes N_{mode} is thus given by the ratio of the bandwidth exhibiting amplification by $c/(2n_{\text{op}} d)$ so that:

$$N_{\text{mode}} = \frac{B}{c/(2n_{\text{op}} d)}$$

Exercise II: Optimization of a multiple quantum well laser diode

1. When considering the optical confinement factor Γ, the lasing oscillation threshold condition of a single QW LD is given by:

$$\Gamma\gamma_{\max} = \text{losses} \;\Rightarrow\; \Gamma\gamma_0 \ln\left(\frac{n_{\text{thr}}}{n_{\text{tr}}}\right) = \alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

2. Knowing that $n_s = (J\tau)/q$ and $n_{\text{tr}} = (J_{\text{tr}}\tau)/q$, we get:

$$\gamma_{\max}(J) = \gamma_0 \ln\left(\frac{J}{J_{\text{tr}}}\right)$$

3. Let us consider an identical coupling of the electromagnetic wave with the wells , we will get $\gamma_{\max,N} = N\gamma_{\max}$, so that:

$$\gamma_{\max,N} = N\gamma_{\max} = N\gamma_0 \ln\left(\frac{J_1}{J_{\text{tr},1}}\right)$$

4. As a result we obtain:

$$J_N = NJ = N\frac{qn_s}{\tau}$$

5. From the previous relationship, we can deduce that:

$$\gamma_{\max,N} = N\gamma_0 \ln\left(\frac{J_N}{NJ_{\text{tr}}}\right)(J_{\text{tr}} = J_{\text{tr},1})$$

6. From the previous relationships, we get:

$$N\Gamma\gamma_0 \ln\left(\frac{J_{N,\text{thr}}}{NJ_{\text{tr}}}\right) = \alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

which leads to:

$$J_{N,\text{thr}} = NJ_{\text{tr}} \exp\left[\frac{1}{N\Gamma\gamma_0}\left(\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right)\right]$$

7. The number of QWs that will minimize the threshold current is deduced from the equation $\mathrm{d}J_N/\mathrm{d}N = 0$:

$$J_{\text{tr}} \exp\left[\frac{1}{N\Gamma\gamma_0}\left(\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right)\right] +$$
$$NJ_{\text{tr}} \exp\left[\frac{1}{N\Gamma\gamma_0}\left(\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right)\right] \times -\frac{1}{N^2\Gamma\gamma_0}\left(\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right) = 0$$
$$\Rightarrow 1 - \frac{1}{N\Gamma\gamma_0}\left(\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right) = 0$$

which leads to:

$$N = \frac{1}{\Gamma\gamma_0}\left(\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right)$$

In fact,

$$N_{\text{opt}} = 1 + \mathbf{I}\left[\frac{1}{\Gamma\gamma_0}\left(\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right)\right]$$

where $\mathbf{I}[...]$ is the integer part of the function enclosed by brackets.

8. The optimum cavity length L_{opt} is obtained through the derivation of $I_{N,\text{thr}}$ with respect to L where $I_{N,\text{thr}} = J_{N,\text{thr}}Lw$, with w the width of the structure. L_{opt} is thus deduced from the equation $\mathrm{d}I_{N,\text{thr}}/\mathrm{d}L = 0$:

$$wJ_{N,\text{thr}} + Lw \times NJ_{\text{tr}} \exp\left[\frac{1}{N\Gamma\gamma_0}\left(\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)\right)\right] \times \frac{1}{N\Gamma\gamma_0}\ln\left(\frac{1}{R_1R_2}\right) \times -\frac{1}{2L^2} = 0$$
$$\Rightarrow wJ_{N,\text{thr}} + Lw \times J_{N,\text{thr}} \times \frac{1}{N\Gamma\gamma_0}\ln\left(\frac{1}{R_1R_2}\right) \times -\frac{1}{2L^2} = 0$$
$$\Rightarrow 1 - \frac{1}{N\Gamma\gamma_0}\ln\left(\frac{1}{R_1R_2}\right) \times \frac{1}{2L} = 0$$
$$\Rightarrow L_{\text{opt}} = \frac{1}{2N\Gamma\gamma_0}\ln\left(\frac{1}{R_1R_2}\right)$$

9.

$$N_{\text{opt}} = 1 + \mathbf{I}\left[\frac{1}{0.1 \times 100}\left(10 + \frac{1}{2 \times 500 \times 10^{-4}}\ln\left(\frac{1}{1 \times 0.32}\right)\right)\right] = 3$$
$$l_{\text{opt}} = \frac{1}{2 \times 10.1 \times 100}\ln\left(\frac{1}{0.32}\right) \approx 570\ \mu\text{m}$$